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An Improved Element Removal Method for Evolutionary Structural Optimization

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The purpose of this study was to develop a new element removal method for ESO (Evolutionary Structural Optimization), which is one of the topology optimization methods. ESO starts with the maximum allowable design space and the optimal topology emerges by a process of removal of lowly stressed elements. The element removal ratio of ESO is fixed throughout topology optimization at 1 or 2%. BESO (bidirectional ESO) starts with either the least number of elements connecting the loads to the supports, or an initial design domain that fits within the maximum allowable domain, and the optimal topology evolves by adding or subtracting elements. But the convergence rate of BESO is also very slow. In this paper, a new element removal method for ESO was developed for improvement of the convergence rate. Then it was applied to the same problems as those in papers published previously. From the results, it was verified that the convergence rate was significantly improved compared with ESO as well as BESO.

Key Words : Improved Element Removal Method, Convergence Rate, Topology Optimization, Evolutionary Structural Optimization

1. Introduction

An important development in topology optimization was made by Bendsφe and Kikuchi (1988) who proposed the homogenization method, in which a material with an infinite number of microscale voids is introduced and the optimization problem is defined by seeking the optimal porosity of a porous medium using an optimality criterion. Some of the results of the homogenization method can be found in the references (Bendsφe, 1989, Suzuki et al., 1991 and Park et al., 1997). Mlejnek et al. (1993) accomplished shape and topology optimization using a simple energy method and a special type of function, that is, Kreisselmeier-Steinhauser function (Kreisselmeier et al., 1979) for calculating effective properties.

Recently, a simple method for shape and topology optimization, called ESO (Evolutionary Structural Optimization), has been proposed by Chu (Chu et al., 1996) and Xie and Steven (Xie et al., 1993 and 1994) which is based on the concept of gradually removing redundant elements of the low stressed part of the material from a structure to achieve an optimal design. Since ESO is accomplished by the fixed element removal ratio of 1 or 2%, the convergence rate may become very slow until an optimum is reached.

And BESO (bidirectional ESO) was suggested by Querin (Querin et al. 1998) for generating the optimum shaped structures. BESO starts with a minimum possible design space, whereas ESO starts with the maximum allowable design space. The structural domain has regions which are heavily under-stressed and regions which are heavily over-stressed. Elements are removed from

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the under-stressed regions and added to the over-stressed regions. But since the element removal ratio and the inclusion ratio of BESO are still small, the convergence rate is very slow.

In this study, a new element removal method for ESO is developed to improve the convergence rate and obtain an optimized design. This method is quite different from BESO. It is an algorithm to determine the removing redundant elements of the low stressed part of a structure, started with the maximum allowable design space. It will be explained in the next section in detail. The validity and efficiency of the improved element removal method (IERM) is verified by comparing the optimized designs for some of the classical optimization problems.

2. The Improved Element Removal Method

The detailed ESO procedure can be found in the work of Xie and Steven(5) and a brief explanation is given in this paper. The strain energy of a structure, which is defined as

$$C=(1/2)\{P\}^T\{u\} \qquad (1)$$

where, $\{P\}$: the nodal load vector and $\{u\}$: the global nodal displacement vector. It is commonly used as the inverse measure of the overall stiffness of the structure. It is obvious that maximizing the overall stiffness is equivalent to minimizing the strain energy. The sensitivity number is defined as

$$\alpha_i=\left(\frac{1}{2}\right)\{u^i\}^T[K^i]\{u^i\} \quad (i=1,\ n) \qquad (2)$$

where, $\{u^i\}$ is the displacement vector of the ith element and $[K^i]$ is the stiffness matrix of the ith element. This indicates the change in the strain energy due to the removal of the ith element. It should be noted that α_i is the element strain energy. In general, when an element is removed, the stiffness of a structure reduces and correspondingly the strain energy increases. To achieve an optimized design, it is obviously most effective to remove the element which has the lowest value of α_i so that the increase in C is minimum.

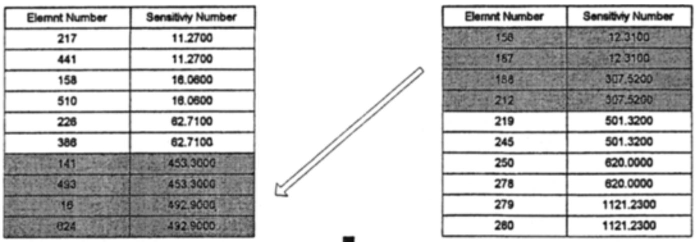
Generally, a structure is not sensitive at the initial stage and becomes sensitive as the number of the removed elements is increased. But, since the removal ratio of ESO is fixed throughout topology optimization at 1 or 2%, it has no flexibility for various types of structures and the convergence rate may not be efficient. In order to improve the convergence rate, it is necessary to increase the element removal ratio at the initial stage and gradually reduce the ratio as the number of the removed elements is increased.

Also, since an optimized design of a structure is highly dependent on the history of element removal, an optimized design is also changed if the removal ratio is changed. But an optimized design should be obtained similarly regardless the element removal ratio.

In order to overcome above two problems, IERM is developed in this study. As mentioned above, when an element is removed the stiffness of a structure reduces and correspondingly the strain energy increases. In the process of ESO, when some elements were removed, the decrease of strain energy often occurred in the next iteration. It means that the removed elements were not properly selected. Thus, before going the next iteration the removed elements from the structure should be determined in order to make the element strain energy increase in the subsequent iteration.

The procedure of IERM is as follows. To start with a piece of material which is large enough to cover the area of the final design of a structure is discretized into a fine mesh of finite elements. Given constraints are applied and a stress analysis is performed. Then the sensitivity numbers are calculated for each element.

In order to determine which elements should be removed from the original structure, the following steps are necessary. First, the numbers of elements with the lowest sensitivity numbers are listed by a flexible element removal ratio of about 7% larger than the 1 or 2% in ESO. Second, the sensitivity numbers are recalculated by another stress analysis for the structure after the listed elements are removed. Third, the numbers of elements having sensitivity numbers which are smaller than the largest sensitivity number of the elements listed at the first step, are listed. Fourth, if the number of

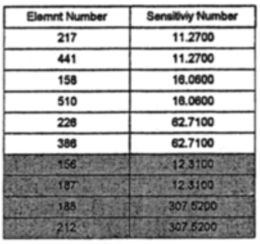


Elemnt Number	Sensitiviy Number
217	11.2700
441	11.2700
158	16.0600
510	16.0600
226	62.7100
386	62.7100
141	453.3000
493	453.3000
16	492.9000
624	492.9000

(a) Elements listed at the first step

Elemnt Number	Sensitiviy Number
156	12.3100
187	12.3100
188	307.5200
212	307.5200
219	501.3200
245	501.3200
250	620.0000
278	620.0000
279	1121.2300
280	1121.2300

(b) Elements listed at the third step



Elemnt Number	Sensitiviy Number
217	11.2700
441	11.2700
158	16.0600
510	16.0600
226	62.7100
386	62.7100
156	12.3100
187	12.3100
188	307.5200
212	307.5200

(c) Elements to be removed

Fig. 1 The procedure of IERM

the elements listed at the third step is smaller than that at the first step, the elements to be removed from the original structure should be all of the elements listed at the third step and the elements with the lowest sensitivity numbers at the first step. Fig. 1 explains this procedure.

If the number of the elements listed at the third step, is larger than that at the first step, the removal ratio should be reduced by 1% and return to the original structure. Whenever this situation occurs, the removal ratio is reduced by 1%. If the removal ratio reduced to 2%, it is fixed by 1% for the following iterations. The reason is that there is no advantage in the convergence rate viewpoint since IERM requires two finite element analyses per iteration. In order to satisfy the required mass of the optimal design, the removal ratio can be reduced to less than 1% at the last iteration.

By using IERM as explained above, a removal ratio of about 7%, which is empirically determined, can be used. Thus, the convergence rate is improved to larger than 50% and also the similar or better optimal designs are obtained compared with the known optimal design for the examples given in the next section.

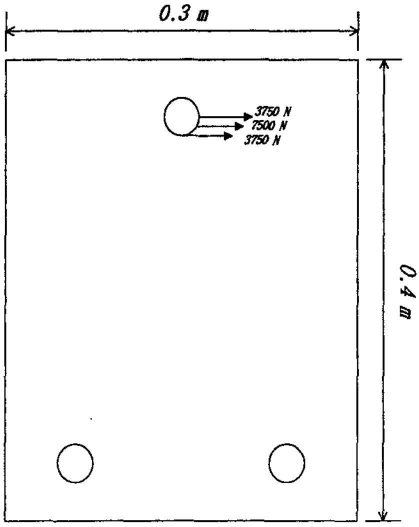


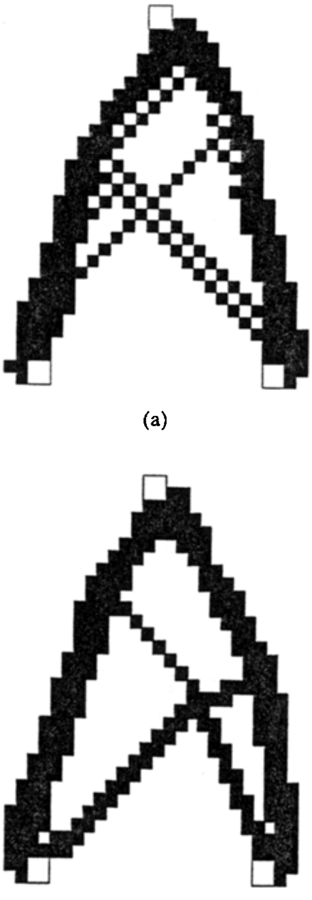
Fig. 2 Initial design conditions of a bracket

3. Application of IERM

IERM was applied to the same example problems as those published previously in order to verify the validity and the effectiveness of it.

3.1 A bracket

A bracket is subjected to three forces of 3.75 kN, 3.75 kN and 7.5 kN as shown in Fig. 2. One


(a)

(b)

Fig. 3 Optimal design obtained by (a) the fixed element removal ratio of 1% and (b) IERM

hole at the top is not fixed and two holes at the bottom are fixed. The thickness is 0.001 m, Young's modulus is 207 GPa and Poisson's ratio is 0.3. The rectangular design domain is discretized into 64×64 quadrilateral elements and the fixed element removal ratio of 1% is applied. IERM with a flexible removal ratio of 7% is applied to the same bracket. The reduction ratio of the optimal design is limited to 80% of the original structure.

The optimal designs by the fixed removal ratio of 1% and by IERM are shown in Fig. 3(a) and (b), respectively. For the optimal design by the fixed removal ratio of 1% the number of iteration is 80, so the call number of finite element analysis is 80. The maximum Mises stress is 1.08E+6 MPa and the maximum displacement is 2. 4 mm. For the optimal design by IERM the number of iteration is 20 and the call number of finite element analysis is 30. The maximum Mises stress and the maximum displacement are the same as those of the fixed removal ratio of 1%. Even though the same maximum Mises stress and the same displacement are obtained by both methods, the optimal designs are obtained differently.

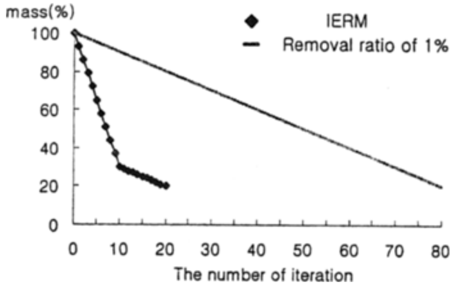


Fig. 4 History of the removal ratio versus the number of iteration

The history graph of removal ratio with respect to the number of iteration is shown in Fig. 4. In the graph, the straight line indicates the history of the fixed removal ratio of 1% with respect to the number of iteration. The bent line indicates the history of IERM with respect to the number of iteration. The first part of the line from the first iteration to the 10th iteration means that the removal ratio of 7% is applied. The second part of the line from the 11th iteration to the 20th iteration means that the removal ratio of 1% is applied. The reason that the removal ratio of 1% is applied after the 10th iteration, is that the removal ratio is reduced to 2% from the removal ratio of 7% because the number of the elements listed at the third step, is larger than that at the first step in the procedure of IERM and such a situation successively occurs five times.

Therefore, the convergence rate based on the call number of finite element analysis is improved to 60.8% and the same optimal design is obtained for the stress and displacement viewpoints under the condition of the reduction ratio limit of 80% of the original structure.

3.2 A Michell type of beam

A Michell type of beam shown in Fig. 5 is subjected to a concentrated force of 20 kN at the

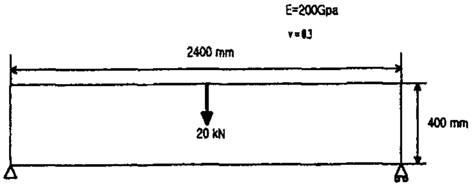


Fig. 5 Initial design conditions of a Michell type of beam

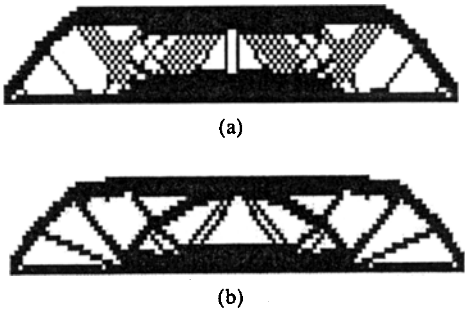


Fig. 6 Optimal designs obtained by (a) the fixed element removal ratio of 1% and (b) IERM

center of the beam. The length and height of the beam are 2.4 m and 0.4 m, respectively. The thickness is 0.001m, Young's modulus is 207 GPa and Poisson's ratio is 0.3. The rectangular design domain is discretized into 120×20 quadrilateral elements and the final reduction ratio of the optimal design is limited to 50.33% of the original structure. The fixed element removal ratio of 1% and IERM with a flexible removal ratio of 7% are applied to the same Michell type of beam. The results obtained by the two methods are compared to each other.

The optimal designs by the fixed removal ratio of 1% and by IERM are shown in Fig. 6(a) and (b), respectively. For the optimal design by the fixed removal ratio of 1% the number of iteration is 51, so the call number of finite element analysis is 51. The last iteration is applied by less than 1% of the removal ratio for requiring mass of the optimal design. The maximum Mises stress is 1. 53E+6 MPa and the maximum displacement is 8. 95 mm. For the optimal design by IERM the number of iteration is 14, the call number of finite element analysis is 20. The last iteration is also applied by less than 1% of the removal ratio for requiring mass of the optimal design. The maximum Mises stress is 1.38E+6 MPa and the maximum displacement is 8.81 mm. The optimal design by IERM is more excellent in both stress and displacement viewpoints.

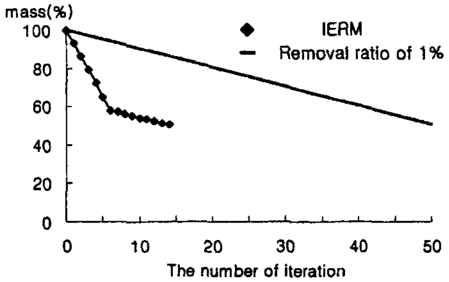


Fig. 7 History of the removal ratio versus the number of iteration

The history graph of removal ratio with respect to the number of iteration is shown in Fig. 7. In the graph, the straight line indicates the history of the fixed removal ratio of 1% with respect to the number of iteration. The bent line indicates the history of IERM with respect to the number of iteration. The first part of the line from the first iteration to the 6th iteration means that the removal ratio of 7% is applied and the second part of the line from the 7th iteration to the 14th iteration means that the removal ratio of 1% is applied. The reason that the removal ratio of 1% is applied after the 6th iteration, is that the removal ratio is reduced to 2% from the removal ratio of 7% because the number of the elements listed at the third step, is larger than that at the first step in the procedure of IERM and such a situation successively occurs five times.

Therefore, the convergence rate is improved to 62.5% based on the call number of finite element analysis and the better optimal design is obtained for the stress and displacement viewpoints under the condition of the reduction ratio limit of 50. 33% of the original structure.

3.3 A short cantilever subjected to bending

A short cantilever shown in Fig. 8 is subjected to a concentrated force of 300 kN at the lower right corner of free end. The length and height of the beam are 0.16 m and 0.10 m, respectively. The thickness is 0.001m, Young's modulus is 207 GPa and Poisson's ratio is 0.3. The rectangular design domain is discretized into 32×20 quadrilateral

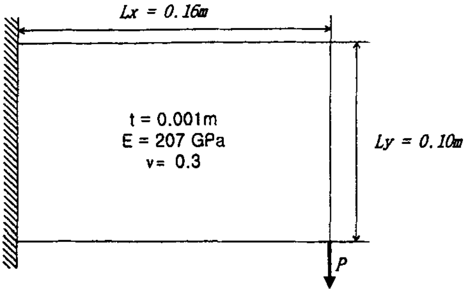
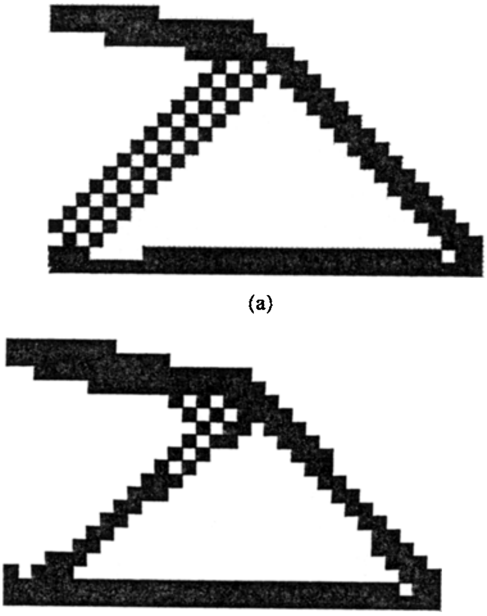


Fig. 8 Initial design conditions of a short cantilever


(a)

(b)

Fig. 9 Optimal designs obtained by (a) the fixed element removal ratio of 1% and (b) IERM

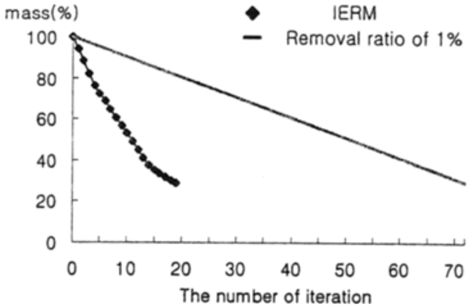


Fig. 10 History of the removal ratio versus the number of iteration

elements and the final reduction ratio of the optimal design is limited to 67% of the original structure. The fixed element removal ratio of about 1% and IERM with a flexible removal ratio of about 6% are applied to the same short cantilever. The results obtained by the two methods are compared to each other.

The optimal designs by the fixed removal ratio of about 1% and by IERM are shown in Fig. 9 (a) and (b), respectively. For the optimal design by the fixed removal ratio of about 1%, the number of iteration is 72, so the call number of finite element analysis is 72. The last iteration is applied by less than 1% of the removal ratio for requiring mass of the optimal design. The maximum Mises stress is 1.84E+6 MPa and the maximum displacement is 1.16 mm. For the optimal design by IERM the number of iteration is 19, the call number of finite element analysis is 33. The maximum Mises stress is 1.83E+6 MPa and the maximum displacement is 1.20 mm. The optimal design by IERM is better in the maximum Mises stress and worse in the maximum displacement viewpoint. But the topology obtained by IERM is better than that by the fixed removal ratio of about 1%.

The history graph of removal ratio with respect to the number of iteration is shown in Fig. 10. In the graph, the straight line indicates the history of the fixed removal ratio of about 1% with respect to the number of iteration. The bent line indicates the history of IERM with respect to the number of iteration. The first part of the line from the first iteration to the 4th iteration means that the removal ratio of about 6% is applied and the second part of the line from the 5th iteration to the 14th iteration means that the removal ratio of 4% is applied. The third part of the line from the 15th iteration to the 19th iteration means that the removal ratio of 1% is applied. The last iteration is applied by less than 1% of the removal ratio for requiring mass of the optimal design. The reason that the removal ratio of 1% is applied after the 4th iteration, is that the removal ratio is reduced to 4% from the removal ratio of 6% because the number of the elements listed at the third step, is larger than that at the first step in IERM procedures and such a situation successively occurs two times. The reason that the removal ratio of 1% is applied after the 14th iteration, is that the removal ratio is reduced to 2% from the removal ratio of 4% because the number of the elements listed at the third step, is larger than that at the first step in the procedure of IERM and such a situation successively occurs two times again.

Therefore, the convergence rate is improved to 54.2% based on the call number of finite element analysis and the similar optimal design is obtained for the stress and displacement viewpoints under the condition of the reduction ratio limit of 67% of the original structure.

4. Conclusions

In this study, IERM was developed in order to improve the convergence rate and the optimal shaped structures. By using the developed IERM, a removal ratio of about 7% larger than the fixed element removal ratio of 1 or 2% in ESO can be used. As a result, the convergence rate is improved to larger than 50% and also similar or better optimal design structures are obtained compared with the results of ESO with the element removal ratio of 1% for some examples in this paper. It is also verified that the removal ratio is applied very flexibly in optimization process.

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